1.0

1.1

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45
50
56
63

2.8 2.5
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3.6
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MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU

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CRC 374 / March 1979

A MODEL FOR ANALYZING A STRAIGHT-RUNNING TORPEDO ATTACK ON A NONEVADING SURFACE SHIP

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AUG 23 1979

Naval Warfare Analysis Group

CENTER FOR NAVAL ANALYSES

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This Research Contribution presents a methodology and a set of computer programs for analyzing an attack on a nonevading surface ship by submarine(s) firing straight-running torpedoes. The model is compatible with ship vulnerability curves and may be used to compute the probability the target sustains a particular level of damage.

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INTRODUCTION

When analyzing engagements between submarines and surface ships, it is often necessary to estimate the effectiveness of the submarine's torpedo attack. Although sophisticated homing torpedoes are being used by many of the world's navies, there continue to be large inventories of straight-running torpedoes. This research contribution presents a methodology and a set of computer programs for analyzing an attack on a nonevading surface ship by submarine(s) firing straight-running torpedoes. It explicitly assumes a triangular horizontal cross section for target ship geometry. We believe this approximation is more applicable to combatants than previously assumed geometries and provides reasonably accurate results for bow and stern shots as well as beam shots. The programs provided make it possible to analyze the effects of several simultaneous, independent submarine attacks. The user must provide some of the torpedo and target ship characteristics and submarine firing doctrine. These are:

- Torpedo
  - speed (knots)
  - maximum run distance (yards)
  - reliability

- Target
  - length at waterline (feet)
  - maximum beam (feet)
  - speed (knots)

- Firing doctrine
  - number of torpedoes in the salvo
  - salvo spread (expressed as a multiple of ship length).

The submarine's position relative to the target is a control variable. The position is expressed as a range and bearing measured clockwise relative to the target's heading.

If a vulnerability curve is available, it is possible to compute the probability that the target sustains a particular level of damage. The David Taylor Ship Research and Development Center, Carderock, is one source that provides vulnerability curves for a ship being put out of action, sunk, or immobilized.

MODELING A TORPEDO ATTACK

The Torpedo Problem

The classical torpedo problem is in three parts:

-1-

(1) Given that a salvo of n torpedoes is fired at a ship, calculate the probability that 0, 1, 2, ..., n torpedo tracks intersect the target if all torpedoes function properly.

(2) Given the results in (1), calculate the probability that 0, 1, 2..., n torpedoes reach the target and explode.

(3) Given that 0, 1, 2, ..., n torpedoes hit the target and explode, calculate the probability that the target is out of action, severely mission degraded, or sunk.

To solve the first part, consider the typical fire control triangle shown in figure 1. It is easily seen from this figure that torpedo lead angle, λ, is

$$\lambda = \text{arc sin} \left[\frac{u}{v} \sin \beta \right] . \tag{1}$$

RA = range to the target at time of firing
RT = lenght of torpedo track
u = target speed
v = torpedo speed
t = running time of torpedo
β = target angle on the bow at time of firing
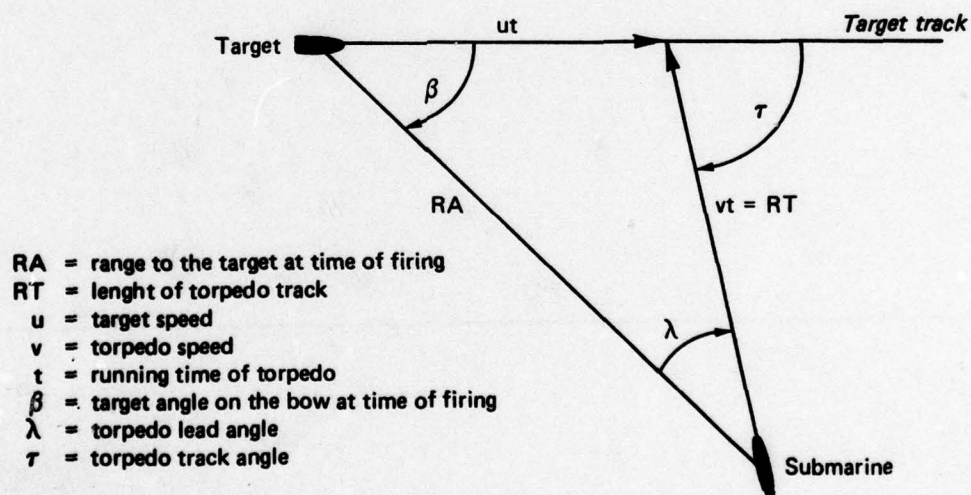λ = torpedo lead angle
τ = torpedo track angle

FIG. 1: TYPICAL FIRE CONTROL TRIANGLE

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Although λ does not depend directly on range, errors in measuring range (to estimate u and β) result in lead angle errors. Lead angle error is derived as:

$$E_{\lambda}^2 = \frac{\left(\dfrac{E_u}{u}\right)^2 \sin^2\beta + E_{\beta}^2 \cos^2\beta}{\left(\dfrac{v}{u}\right)^2 - \sin^2\beta} + E_i^2 \tag{2}$$

where:

E_{λ} = standard deviation of the lead angle error,

E_u = standard deviation of the errors in estimating u ,

E_{β} = standard deviation of the errors in estimating β ,

E_i = standard deviation of incidental errors.

Values for these standard deviations have been estimated in reference 1 from short-range (less than 4,000 yards) attacks on Japanese ships during World War II as follows:

$\dfrac{E_u}{u} = 0.15$,

E_{β} = 10 degrees (0.1745 radians), and

E_i = 0.05, 0.03, and 0.015 radians for 1,000, 2,000, and 3,000-yard ranges to the target at the time of firing.

Other values of E_i , where needed, are taken from the continuous function:

$$E_i = \exp\{-0.0006 \ RA - 2.4\} , \tag{3}$$

where RA = firing range in yards.

These parameters are used in the computer program presented later. Data from more recent experience with straight-running torpedoes can be easily inserted into the program, if it is available.

The Model

E_{λ} is used to estimate torpedo hit probabilities, assuming that the torpedo track has a Gaussian (normal) distribution about its intended course with mean zero and standard deviation E_{λ} . The probability of a hit on a ship by a single torpedo aimed at the center of the target is the integral from $-L_{eff}/2E_{\lambda}RT$ to $+L_{eff}/2E_{\lambda}RT$ of the standard normal distribution. L_{eff} is the effective length of the target measured perpendicular to the torpedo track, and RT is the length of the torpedo track.

-3-

For a salvo of torpedoes, the limits of integration for 0, 1, 2, or more hits are a function of $L_{eff}/E_\lambda RT$, the spread of the salvo, and the number of torpedoes in the salvo. When a salvo of more than one torpedo is fired at a target, it will be assumed that the center of the salvo is fired at the center of the ship, the torpedoes are equally spaced, and the individual torpedoes within the salvo run in parallel with no error between torpedoes. The only error, E_λ, is the placement of the center of the salvo on the target.

A formula for E_λ was presented in equation 2. From figure 1, it is apparent that $RT = vt$. We next need to determine L_{eff}.

Target Ship Geometry

To obtain an exact formula for L_{eff} requires a description of the horizontal cross section of the target ship at torpedo running depth. This is impractical. Prior models generally use approximations based only on length, with the beam assumed to be zero. This produces severe errors for bow and stern shots. Often implicitly rather than explicitly, some geometric shape is assumed for the waterline cross section. In many cases, the shape is a rectangle of length L and width B, clearly applicable to barges and to a lesser extent merchant vessels and auxiliaries, but not to combatants. Modern combatant hulls tend to have fine lines at the bow and nearly uniform width aft of the maximum beam.

We considered several geometric shapes. For example, the combination of a rectangle plus an isosceles triangle provides an excellent approximation, but a combination of geometric shapes complicates the mathematical formulation considerably. In general, compound shapes result in complex mathematics. A triangle alone, however, provides results that are nearly as good with much simpler mathematics. Therefore, we used an approximation based on a triangular shape, with the small angle pointed forward and the base as wide as the ship's beam aft (see figure 2). This explicitly assumed triangular ship geometry is the basis for the computation of L_{eff} in this research contribution and the computer program.

A triangular configuration provides a reasonable approximation of the shape of most modern combatants in the critical locations near the bow and stern. The small errors that result from using this configuration take effect principally at small non-zero torpedo angles and to a lesser degree for near-180 degree stern shots. Typical beam-to-length ratios result in an angle at the bow of 3-1/2 to 4 degrees relative to the ship's centerline. A modern combatant has an angle of approximately 10 to 12 degrees. With a track angle between these values, the model understates hits at the bow. Except at track angles of exactly 0, 90, and 180 degrees, the model overstates hits at the stern. These
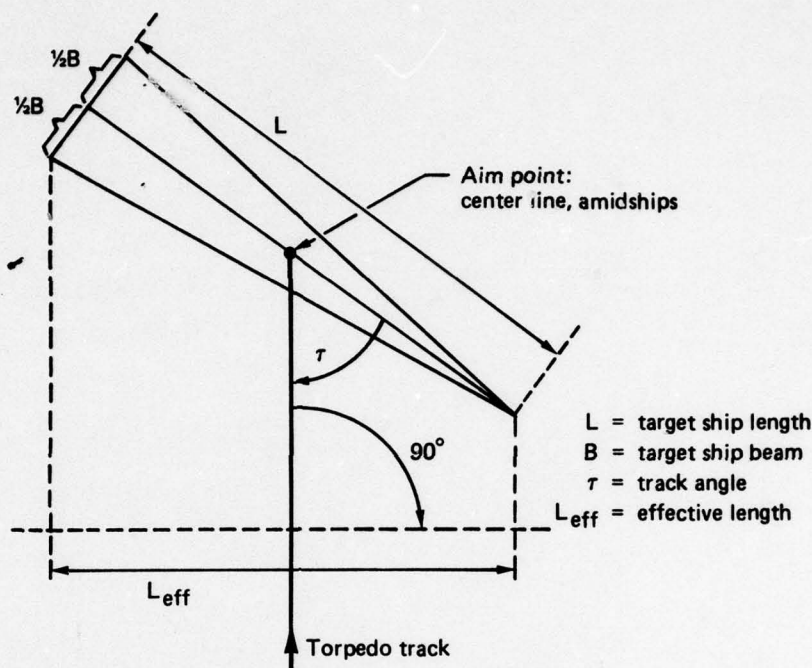
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FIG. 2: ASSUMED SHIP GEOMETRY

errors partially compensate for one another. The model is correctly biased to the extent that torpedoes traveling at a high relative velocity at angles of 3 to 12 degrees relative to ship's track would be subject to hydrodynamic forces at the bow and/or not detonate because the angle of incidence relative to hull plating is very small.

The mathematical approximation of L_{eff} for a triangular shape, including a first-order adjustment for relative motion, is:

$$L_{eff} = \left[\max \left\{ \begin{array}{l} L \times |\sin \tau| \\ \frac{B}{2} \times |\frac{u}{v} + \cos \tau| \end{array} \right\} + \frac{B}{2} \times |\frac{u}{v} + \cos \tau| \right] \div \left(1 + \frac{u}{v} \cos \tau \right) \quad (4)$$

This approximation is used in the APL computer program described later.

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Torpedo Salvos

To develop the limits of integration for multiple hits on a target, the effective length of the target is converted to an angular measure (the angle subtended from the aim point when the torpedo is at the end of its intended run, L_{eff}/RT) and defined as a ratio to the standard deviation of the lead angle error: $TL = L_{eff}/E_\lambda RT$. The coverage factor, CF, is expressed as a multiple of TL.

It will be necessary to integrate the standard normal distribution between some limits a_i and b_i, to compute the probability of i tracks intersecting the target. From this, the probability of h hits can be calculated using torpedo reliability.

The limits of integration, a_i and b_i, can be represented algebraicly in terms of TL, CF, and n (number of torpedoes in the salvo). For convenience we will also define the spacing between torpedoes as $SP = (CF \times TL) / (n-1)$.

Under the assumptions of this model, a target is missed (no track intersections), when the center of the salvo is displaced from the aimpoint by at least $TL/2 + 1/2$ the salvo width; that is, $(CF + 1) \times TL/2 = a_0$, $\infty = b_0$ (see figure 3).
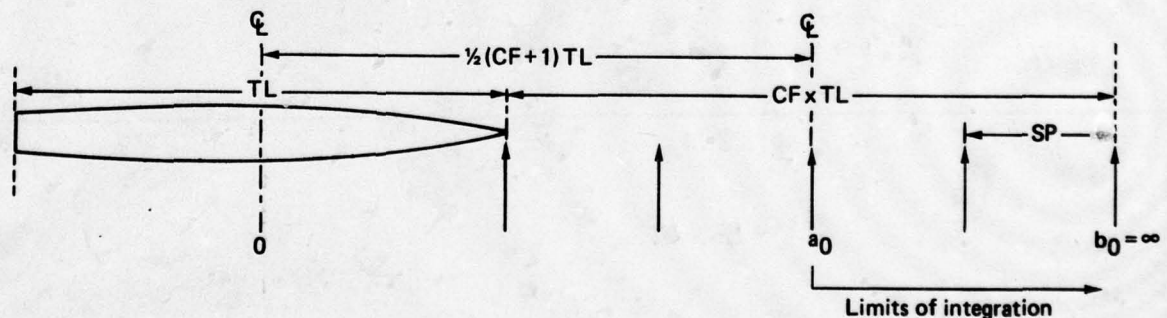

FIG. 3: LIMITS OF INTEGRATION: 5 TORPEDOES, NO HITS

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We will get one track intersection if the center of the salvo is displaced from target center less than $a_0 = b_1$ and more than $a_0 - SP = a_1$ (see figure 4). Similarly, we will get two track intersections if the center of the salvo is displaced less than $b_2 = a_1$, but more than $a_2 = a_1 - SP = a_0 - 2 \times SP$ (see figure 5). This leads to the general form:

$$a_i = a_0 - i \times SP;$$

$$b_i = a_{i-1} = a_0 - (i - 1) \times SP.$$

Since there are only $n-1$ spaces, $0 < i \leq n-1$. However, these values are not exactly correct because of two limitations:

<u>First:</u> If the coverage factor is greater than 1 (i.e., the salvo spread is wider than the effective length of the ship), then a point will come at which $a_i > 0$ but $i \times SP > TL$.

That is, torpedoes will miss both ahead and behind the target ship (see figure 6). Beyond this point, the number of track intersections alternates between i and $i-1$. This fluctuation can be visualized by thinking of the center of the salvo moving from bow to stern across the target. At some point, a torpedo "falls off" the stern (there are now $i-1$ intersections) and then another "comes on" at the bow (i intersections), and so on. After the salvo center passes the target's mid point, the number of intersections will continue to fluctuate for a while and then diminish until eventually no tracks intersect the target.

The limits of integration are handled in the following manner. When $i \times SP > TL$ for the first time, the limits of integration for i track intersections are:

$$a_i = a_0 - TL \text{ and } b_i = a_{i-1} \qquad .$$

After the point when the center of the salvo is at $a_0 - TL$, we get one less track intersection because one torpedo misses astern as the salvo moves aft. As the salvo continues to move aft, the next torpedo "comes on" at the bow when the salvo center is at $a_0 - (i) \times SP$ (recall $i \times SP > TL$). Thus integration between the limits $a_0 - (i) \times SP$ and $a_0 - TL$ contributes to the probability of $(i-1)$ hits.

As the salvo moves aft, we get i hits until the second torpedo "falls off" the stern when the salvo center is at $a_0 - (TL + SP)$. We then get $i-1$ hits until another "comes on" at the bow when salvo center is at $a_0 - (i + 1)SP$. Continuing in this pattern, we see that for $i > (TL \div SP)$, we end up with alternate intervals contributing to the probability of i and $i-1$ hits.
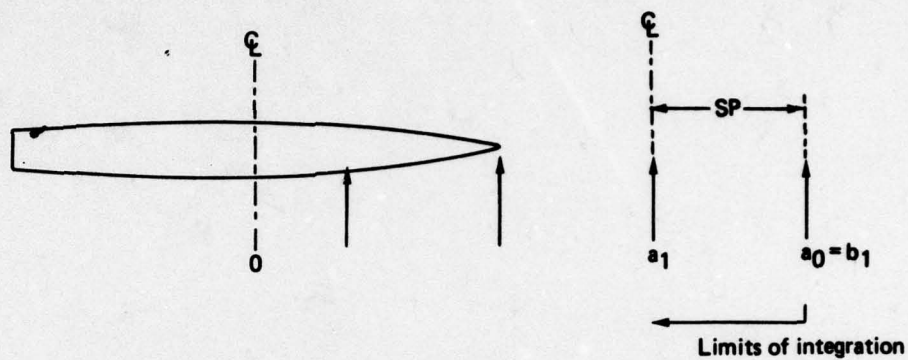
FIG. 4: LIMITS OF INTEGRATION: 5 TORPEDOES, 1 HIT
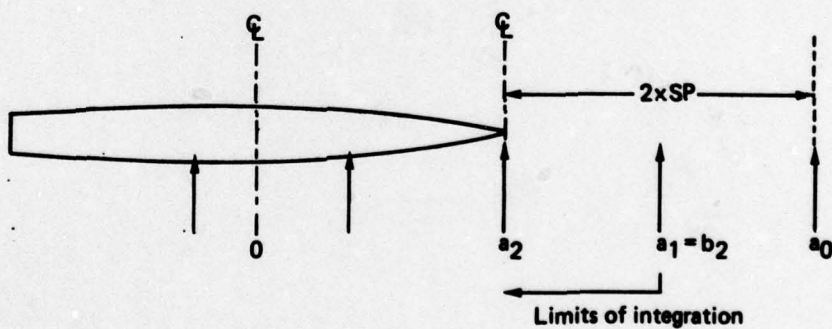
FIG. 5: LIMITS OF INTEGRATION: 5 TORPEDOES, 2 HITS

Two hits

a_2 $a_1 = b_2$ a_0

2SP

Limits of integration

Three hits

a_3 $a_2 = b_3$ a_1 a_0

TL

0

Limits of integration
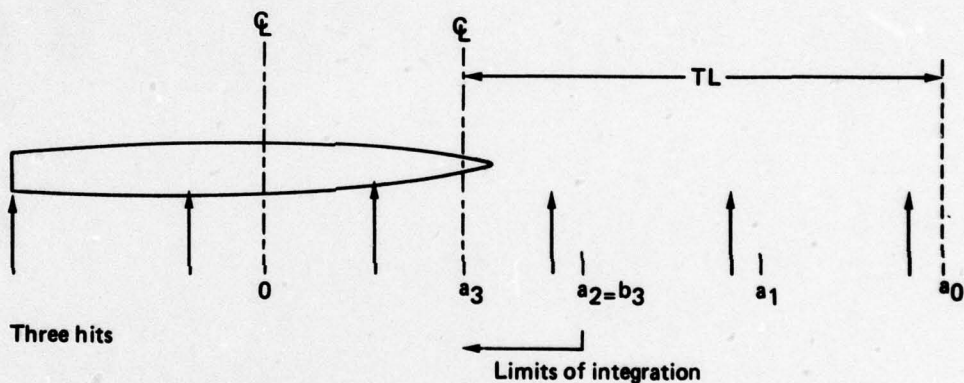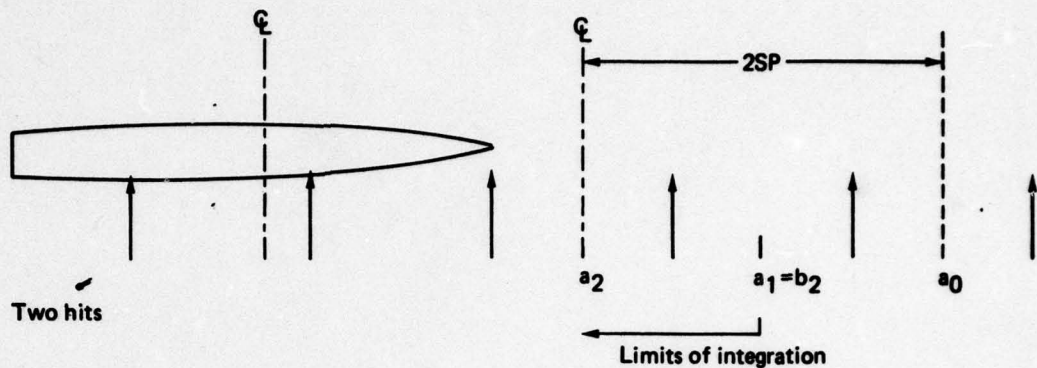
Note: $3 \times SP > TL$, so $a_3 = a_0 - TL$ not $a_0 - 3SP$

Two hits

a_4 $a_3 = b_4$ a_2 a_1 a_0

3SP

Limits of integration
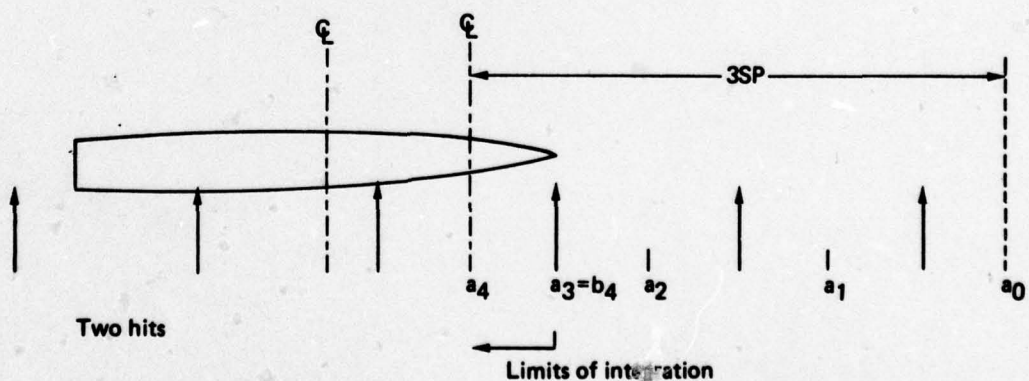
Note: Misses to both left and right of target

FIG. 6: INTEGRATION LIMITS FOR CF>1: 6 TORPEDOES, CF = 1.8

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Intervals of the form $a_0 - (i + j)SP$ to $a_0 - (TL + (j)SP)$ contribute to the probability of $i-1$ hits.

Intervals of the form $a_0 - (TL + (j)SP)$ to $a_0 - (i-1 + j)SP$ contribute to the probability of i hits.

That is

$$\text{Prob of } i \text{ hits} = \int_{a_0 - (TL + (0)SP)}^{a_0 - (i-1+0)SP} N(0,1) + \int_{a_0 - (TL + (1)SP)}^{a_0 - (i-1+1)SP} N(0,1) + \int_{a_0 - (TL + (2)SP)}^{a_0 - (i-1+2)SP} N(0,1) + \ldots$$

and

$$\text{Prob of } i-1 \text{ hits} = \int_{a_0 - (i+0)SP}^{a_0 - TL + (0)SP} N(0,1) + \int_{a_0 - (i+1)SP}^{a_0 - (TL + (1)SP)} N(0,1) + \int_{a_0 - (i+2)SP}^{a_0 - (TL + (2)SP} N(0,1) + \ldots$$

where $N(0,1)$ is the standard unit normal distribution. These are still not exactly correct values. Each series must be stopped after a limited number of terms. This leads to the second limitation.

<u>Second</u>: We will eventually multiply the derived probabilities by 2 because the aim point was assumed to be the center of the target and the standard normal distribution is symmetric. This means we must integrate only over intervals to one side or the other of target center (see figure 7).

A check must be made at each step to insure the limits of integration are all non-negative.

The smallest integer value of i for which $i \times SP \geq TL$ is an upper limit on the number of intersecting tracks, along with n (the number of torpedoes in the salvo). The same constraint can also be expressed as follows: the maximum number of tracks is the smallest integer i, such that $i \geq n/CF$ and $i \leq n$.

Reliable Hits on a Target

Once the probabilities of $0, 1, 2, \ldots, n$ intersecting tracks are computed as previously described, the next step is to determine the probabilities of $0, 1, 2, \ldots, n$ reliable torpedo hits. To make a hit, a torpedo must travel an intersecting track and perform reliably. If we use the symbol $P(T=i)$ for the probability that there are i intersecting tracks and θ for the reliability of an individual torpedo, then the probability of h reliable hits $P(H=h)$ where $h \leq i$ can be computed as:

$$P(H=h) = \sum_{i=h}^{n} P(T=i) \times P(H=h \mid T=i) . \tag{5}$$

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Two hits

Limits of integration

Three hits

Limits of integration
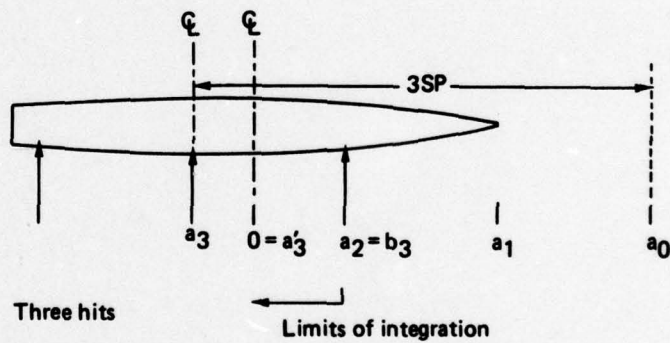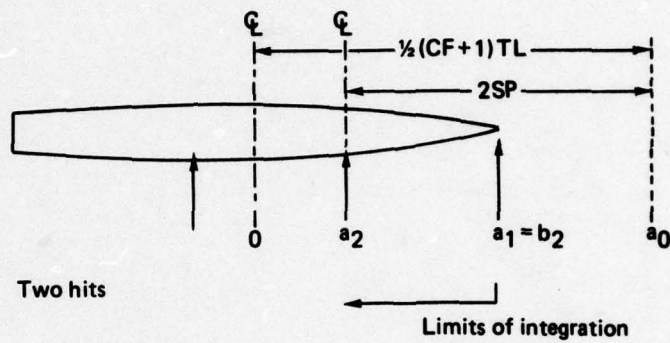
Note: $a_3 < 0$ so $a'_3 = 0$

FIG. 7: INTEGRATION LIMITS FOR $i \times SP > \frac{1}{2}(CF+1)TL$

Assuming that the reliability of one torpedo is independent from that of another, we can use a binominal distribution for $P(H=h \mid T=i)$, and then

$$P(H=h) = \sum_{i=h}^{n} P(T=i) \left\{ \binom{i}{h} (1-\theta)^{(i-h)} \theta^h \right\} \quad .$$ (6)

Target Vulnerability

The probability that a ship receives a particular level of damage is the probability of a particular level of damage given h reliable hits times the probability of h reliable hits summed over the range of possible values for h. If we symbolize the vulnerability vector providing the probabilities of damage given h hits as $V(h)$, $h=1, 2, \ldots, n$, we can compute the probability of damage (PD) to a ship by the formula

$$PD = \sum_{h=1}^{n} P(H=h) \times V(h) \quad .$$ (7)

The damage probability can also be interpreted as the expected fraction of attacked ships in a force that are damaged.

Simultaneous Attacks

To properly account for simultaneous, independent attacks by more than one submarine, the probabilities of reliable hits for each submarine must be combined into a probability of reliable hits by all submarines, before performing the step indicated in equation 7 above. For example, when there are two submarines, we first compute $P_1(H=h)$ and $P_2(H=h)$ for each submarine. Then

$$P(H=h) = \sum_{i=0}^{h} P_1(H=i) \times P_2(H=h-i) \quad .$$ (8)

Although equation 8 assumes attacks by only two submarines, it can be extended to any arbitrary number of simultaneous, independent attacks.

COMPUTER PROGRAM

The APL computer programs and subroutines in appendix A provide a variety of techniques for computing intersecting tracks, hits, and/or damage probabilities for one or more attacking submarines.

The first program listed, and the simplest to use, is Program ATTACK. This program is limited to computing results for attacks by one submarine; it requires only limited knowledge of the program and APL. ATTACK is a conversational program that

computes and displays intersecting track probabilities, hit probabilities, and/or damage probability. It also displays, on request, the results of such intermediate calculations as track angle, torpedo run time and distance, effective ship length, and the value of TL. Intersecting tracks and hits are displayed as probability vectors (the probability of 0, 1, 2, ...n tracks or hits). Appendix B shows a sample ATTACK run in which user inputs are underlined.

Appendix A also contains 7 additional programs that do the same computations as program ATTACK, but with more flexibility. These programs permit the analysis of more complex tactical situations (i.e., multiple submarines). These programs are more complex than ATTACK, however, and familiarity with APL is essential to use them effectively. The basic use of each of the programs is shown in table 1. Several examples are shown in appendix C.

TABLE 1

TORPEDO ATTACK PROGRAMS

PROGRAM	USE	INPUT REQUIRED	OUTPUT
ATTACK	Computes intersecting tracks, hits and damage probability for one submarine	Response to conversational inquiry	Labeled displays as requested
TRACKS	Computes probabilities of 0,1,2,...,n intersecting tracks	A vector of target, torpedo and tactical parameters. See page C-1.	A vector of intermediate calculations, and intersecting track probabilities. See page C-1.
HITS	Computes probabilities of 0,1,2,...n hits	Track intersection probabilities (part of the output of TRACKS) plus torpedo reliability	A vector of hit probabilities. See page C-2.
ELOSS	Computes probability of damage	Output of HITS (or SUBS or COMBINE) plus a vulnerability vector (probability) of damage given 0,1,2,... hits[a]). See page C-2.	The probability of damage (expected fraction of target ships damaged)
SUBS	Computes a track or hit probability vector for two or more submarines firing from the same or from symmetric positions, or for two or more salvos from the same (relative) position	The output of HITS or the probability vector output of TRACKS and number of submarines (or salvos).	The probability vector for 0,1,2,...z hits or tracks where z=number of submarines (or salvos) times the size of salvo
COMBINE	Combines two[b] probability vectors from TRACKS or HITS into a single probability vector (to combine hit probabilities for non-identical attacks)	Two probability vectors of length L_1 and L_2	One probability vector of length $L_1 + L_2 - 1$
BINAPPROX	Computes a vector of binomal probabilities (by normal approximation if necessary)	N,P -- the binomal parameters	A probability vector of length N+1
NORM	Computes areas under the standard (unit) normal curve by approximation	One or more values representing distances from the mean measured in standard deviations	A left-hand cumulative normal probability for each input

[a] If the vulnerability vector does not contain probabilities for all possible numbers of hits, it is extended by projecting a straight line through the last two values given until reaching an upper bound of 1.0.

[b] COMBINE can be chained to handle a combination of more than two vectors.

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REFERENCES

1. Center for Naval Analyses, Submarine Operations Research Group, Research Report NR 2, "Effect of Firing Position on the Success of Torpedo Attacks," Unclassified, Mar 1944

APPENDIX A

APL COMPUTER PROGRAMS

```
      ∇ ATTACK;X;SW;HEAD;R;VUL
[1]    'WHAT IS TARGET SHIP LENGTH IN FEET?'
[2]    X←□
[3]    'WHAT IS TARGET SHIP BEAM IN FEET?'
[4]    X←X,□
[5]    'WHAT IS TARGET SHIP SPEED IN KNOTS?'
[6]    X←X,□
[7]    'WHAT IS RANGE TARGET TO SUB IN YARDS?'
[8]    X←X,□
[9]    'WHAT IS ASPECT ANGLE (ANGLE ON THE TARGET BOW) IN DEGREES?'
[10]   X←X,□
[11]   'HOW MANY TORPEDOS IN A SALVO?'
[12]   X←X,□
[13]   'WHAT IS THE COVERAGE FACTOR?  (TORPEDO SPREAD ÷ TARGET SHIP
       LENGTH)'
[14]   X←X,□
[15]   'WHAT IS TORPEDO SPEED IN KNOTS?'
[16]   X←X,□
[17]   'WHAT IS TORPEDO MAXIMUM RANGE IN YARDS?'
[18]   X←X,□
[19]   'YOUR ANSWERS IN ORDER WERE:',□R;X;□R,'ARE THEY OK?'
[20]   →1×ι∧/'NO   '=5↑□
[21]   'DO YOU WANT TRACK PROBABILITIES DISPLAYED?'
[22]   SW←∧/'YES  '=5↑□
[23]   'DO YOU WANT TRACK ANGLE, RUN DISTANCE, RUN TIME, EFFECTIVE
       LENGTH, AND TL DISPLAYED?'
[24]   SW←SW,∧/'YES  '=5↑□
[25]   'DO YOU WANT HITS DISPLAYED?'
[26]   SW←SW,∧/'YES  '=5↑□
[27]   'IF YOU HAVE A VULNERABILITY VECTOR I''LL COMPUTE EXPECTED
       LOSSES.'
[28]   'INPUT THE VECTOR STARTING WITH PROB OF LOSS GIVEN ONE HIT,PROB
       OF LOSS GIVEN TWO HITS, ...ETC, IF AVAILABLE. OTHERWISE HIT THE
       RETURN.'
[29]   →0×ι0=+/SW←SW,1≠ρVUL←,⍎'0 ',100↑□
[30]   →L1×ι0=+/SW[3,4]
[31]   'I''LL NEED TORPEDO RELIABILITY:'
[32]   R←□
[33] L1:TEMP←TRACKS X
[34]   →NOSHOT×ι1=∧/(180,(2ρ10*7),0,0,1)=6↑TEMP
[35]   →L2×ιSW[1]=0
[36]   'TRACK PROBABILITIES',□R,((5 0⍴0),10 0⍕ιX[6]),□R,10 5⍕5↑TEMP
[37] L2:→L3×ιSW[2]=0
[38]   'TRACK ANGLE   RUN DISTANCE    RUN TIME      EFFECTIVE SHIP
       LENGTH',□R,' (DEGREES)       (YARDS)         (SECONDS)      (FEET)
        TL(STD.DEV.)'
[39]   7 1 15 0 15 1 13 0 15 4⍕5↑TEMP
[40] L3:→L4×ιSW[3]=0
[41]   'HIT PROBABILITIES',□R,((5 0⍴0),10 0⍕ιX[6]),□R,10 5⍕R HITS 5↓
       TEMP
[42] L4:→0×ιSW[4]=0
[43]   'EXPECTED LOSS PROBABILITY:   ';VUL ELOSS R HITS 5↓TEMP
[44]   →0
[45] NOSHOT:'THERE IS NO CHANCE OF A HIT GIVEN THIS SET OF DATA.'
      ∇
```

```
      ∇ Z←TRACKS X;SL;SB;SS;RA;AN;NT;CF;TS;MR;LA;TL;RT;K;SP;M;T;E;I;EL
[1]    →0×⍳9≠⍴X
[2]    SL←X[1]
[3]    →NOSHOT×⍳0=SL+SB←X[2]
[4]    SS←X[3]
[5]    →NOSHOT×⍳0=RA←X[4]
[6]    AN←○X[5]÷180
[7]    →NOSHOT×⍳0=NT←X[6]
[8]    CF←X[7]
[9]    →NOSHOT×⍳0=TS←X[8]
[10]   →NOSHOT×⍳0=MR←X[9]
[11]   →NOSHOT×⍳1<(SS÷TS)×1○AN
[12]   LA←¯1○(SS÷TS)×1○AN
[13]   →NOSHOT×⍳0≥T←1+(SS÷TS)×2○AN+LA
[14]   Z←180×(AN+LA)÷○1
[15]   →L0×⍳SS=0×E←*¯2.4-0.0006×RA
[16]   E←((E*2)+((0.0225×(1○AN)*2)+0.03045025×(2○AN)*2)+((TS÷SS)*2)-(1○
       AN)*2)*0.5
[17]   L0:→L1×⍳0=1|AN÷○1+0×RT←RA÷T
[18]   RT←RA×(1○AN)÷1○(○1)-AN+LA
[19]   L1:→NOSHOT×⍳MR<RT
[20]   EL←(T+(SL×|1○AN+LA)⌈T+SB×0.5×|(SS÷TS)+2○AN+LA)÷1+(SS÷TS)×2○AN+LA
[21]   Z←Z,RT,(RT÷TS×0.5626032861),EL,TL←EL÷E×3×RT
[22]   →ONE×⍳(NT=1)∨CF=0
[23]   M←(1+CF)×TL÷2
[24]   SP←(CF×TL)÷NT-1
[25]   →CFL×⍳CF≤1
[26]   K←LTL÷SP
[27]   X←NORM M,M-SP×⍳K
[28]   X←(1,X)-X,NORM M-TL
[29]   I←0
[30]   AGAIN:X[¯1+⍴X]←X[¯1+⍴X]-(NORM T←0⌈M-SP×K+I←I+1)-NORM M-TL+I×SP
[31]   →END×⍳T=0
[32]   X[⍴X]←X[⍴X]+(NORM T)-NORM 0⌈M-TL+I×SP
[33]   →END×⍳0≥M-TL+I×SP
[34]   →AGAIN
[35]   CFL:K←NT-1+CF=1
[36]   X←NORM M,M-SP×⍳K
[37]   X←(1,X)-X,0.5
[38]   END:Z←Z,(NT+1)↑2×X
[39]   →0
[40]   ONE:Z←Z,(1-SP),((NT-1)⍴0),SP+2×(NORM TL÷2)-0.5
[41]   →0
[42]   NOSHOT:Z←(6+NT)↑180,(2⍴10*7),0,0,1
      ∇
```

```
      ∇ Z←P HITS V;N;I
[1]   N←ρV
[2]   Z←N↑V[I←1]
[3]   AGAIN:Z←Z+N↑V[I+1]×BINAPPROX I,P
[4]   →AGAIN×ιN>I←I+1
      ∇



      ∇ R←VUL ELOSS PROB
[1]   →(L1,L2,L3)[2+×(ρVUL)-ρPROB]
[2]   L1:→L2×ι1=VUL[ρVUL]
[3]   →L3×ι0≠ρVUL←(ρPROB)↑1⌊VUL,VUL[ρVUL]+(VUL[ρVUL]-VUL[¯1+ρVUL])×ιρ
      PROB
[4]   L2:VUL←(ρPROB)↑VUL,(ρPROB)ρ1
[5]   L3:R←+/VUL×PROB
      ∇



      ∇ R←N SUBS P
[1]   →0×ι(N←⌊N)<1,0ρR←1 0
[2]   →0×ιN=1,0ρR←P
[3]   R←P COMBINE R
[4]   →3×ι1<N←N-1
      ∇



      ∇ R←P COMBINE Q
[1]   R←+/[1](0,¯1↑¯1+ρP)⌽(P∘.×Q),('(0,¯1)+ρP)ρ0
      ∇



      ∇ V←BINAPPROX Y;X;N;P;M;VAR
[1]   →0×ι(P≥1)∨P≤0,0ρV←(N+1)↑(1≤P←Y[2])↓1,(NρO),1,X←0,ιN←⌊Y[1]+0.5
[2]   →3+(N>200)∨(N>50)∧N>25×(P÷1-P)⌈(1-P)÷P
[3]   →0×V←(P*X)×(X!N)×(1-P)*N-X
[4]   V←((VAR×○2)*¯0.5)×*-((X-M)*2)÷2×VAR←(1-P)×M←N×P
      ∇



      ∇ P←NORM X;N
[1]   N←1,P←0ρX←,X
[2]   P←P,1-((÷(2×○1)*0.5)×*-0.5×X[N]*2)×+/0.31938153 ¯0.356563782
      1.781477937 ¯1.821255978 1.330274429×(÷1+0.2316419×X[N])*ι5
[3]   →2×ι(ρX)≥N←N+1
      ∇
```

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APPENDIX B

EXAMPLE OF USE OF THE INTERACTIVE PROGRAM ATTACK

Note: Underlines indicate user entries.

```
     ATTACK
WHAT IS TARGET SHIP LENGTH IN FEET?
□:
     1000
WHAT IS TARGET SHIP BEAM IN FEET?
□:
     120
WHAT IS TARGET SHIP SPEED IN KNOTS?
□:
     20
WHAT IS RANGE TARGET TO SUB IN YARDS?
□:
     3500
WHAT IS ASPECT ANGLE (ANGLE ON THE TARGET BOW) IN DEGREES?
□:
     135
HOW MANY TORPEDOS IN A SALVO?
□:
     6
WHAT IS THE COVERAGE FACTOR?   (TORPEDO SPREAD ÷ TARGET SHIP LENGTH)
□:
     2.5
WHAT IS TORPEDO SPEED IN KNOTS?
□:
     50
WHAT IS TORPEDO MAXIMUM RANGE IN YARDS?
□:
     10000
YOUR ANSWERS IN ORDER WERE:
1000   120  20   3500   135  6  2.5   50   10000
ARE THEY OK?
YES
DO YOU WANT TRACK PROBABILITIES DISPLAYED?
YES
DO YOU WANT TRACK ANGLE, RUN DISTANCE, RUN TIME, EFFECTIVE LENGTH, AND
     TL DISPLAYED?
YES
DO YOU WANT HITS DISPLAYED?
YES
IF YOU HAVE A VULNERABILITY VECTOR I'LL COMPUTE EXPECTED LOSSES.
INPUT THE VECTOR STARTING WITH PROB OF LOSS GIVEN ONE HIT,PROB OF LOSS
     GIVEN TWO HITS, ...ETC. IF AVAILABLE. OTHERWISE HIT THE RETURN.
.5  .75    1
I'LL NEED TORPEDO RELIABILITY:
□:
     .85
TRACK PROBABILITIES
     0          1          2          3          4          5          6
  0.20017    0.16000    0.63983    0.00000    0.00000    0.00000    0.00000
TRACK ANGLE    RUN DISTANCE     RUN TIME      EFFECTIVE SHIP LENGTH
 (DEGREES)       (YARDS)       (SECONDS)        (FEET)    TL(STD.DEV.)
  151.4          5175            184.0           781         0.7320
HIT PROBABILITIES
     0          1          2          3          4          5          6
  0.23857    0.29915    0.46228    0.00000    0.00000    0.00000    0.00000
EXPECTED LOSS PROBABILITY:  0.4962868154
```

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APPENDIX C

EXAMPLES OF APL PROGRAM USE

APPENDIX C
EXAMPLES OF APL PROGRAM USE

Example 1: Torpedo tracks (program TRACKS)

Maximum torpedo range (yards) ────────────────┐
Torpedo speed (knots) ──────────────────────┐ │
Coverage factor ──────────────────────────┐ │ │
Number of torpedoes in salvo ───────────┐ │ │ │
Angle on the target bow (degrees) ─────┐ │ │ │ │
Range to target (yards) ─────────────┐ │ │ │ │ │
Target speed (knots) ──────────────┐ │ │ │ │ │ │
Target beam (feet) ──────────────┐ │ │ │ │ │ │ │
Target length (feet) ──────────┐ │ │ │ │ │ │ │ │

```
        TRACKS 1000   120   20   3500   135   6   2.5   50   10000
151.4299402     5175.037571     183.9675558     781.4423676   0.7320379418
        0.2001704761   0.1599962003   0.6398333226   0   0   0   0
```

Probability of:

0 intersecting tracks ───────┐
1 intersecting tracks ─────────┐
2 intersecting tracks ───────────┐
3 intersecting tracks ─────────────┐
4 intersecting tracks ───────────────┐
5 intersecting tracks ─────────────────┐
6 = n intersecting tracks ──────────────┐

{ The first values in the output are:
  Track angle (degrees); run distance (yards), run time (seconds),
  effective ship length (feet), and TL, respectively. }

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EXAMPLES OF APL PROGRAM USE

Example 2: Torpedo hits and tracks (programs HITS and TRACKS)

Torpedo reliability

To remove intermediate values from TRACKS output

Same as input for TRACKS, example 1

```
.85 HITS 5↓   TRACKS   1000 120   20   3500   135   6   2.5   50   10000
0.2385661559   0.2991542675   0.4622795756   0   0   0   0
```

Probability of:
- n = 6
- 5 hits
- 4 hits
- 3 hits
- 2 hits
- 1 hit
- 0 hits
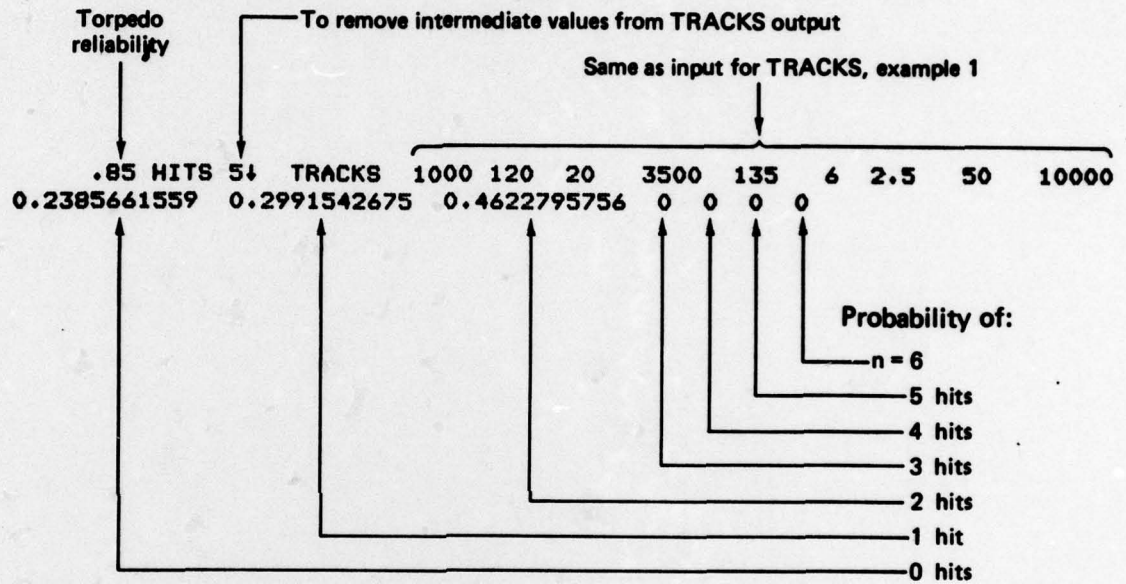
Example 3: Expected losses from torpedoes (programs ELOSS, HITS, and TRACKS)

Probability of damage given 0 hits
Probability of damage given 1 hit
Probability of damage given 2 hits
Probability of damage given 3 hits

Same as above, example 2

```
0 .5 .75 1 ELOSS .85 HITS 5↓ TRACKS 1000 120 20 3500 135 6 2.5 50 10000
0.4962868154
```

Probability of damage
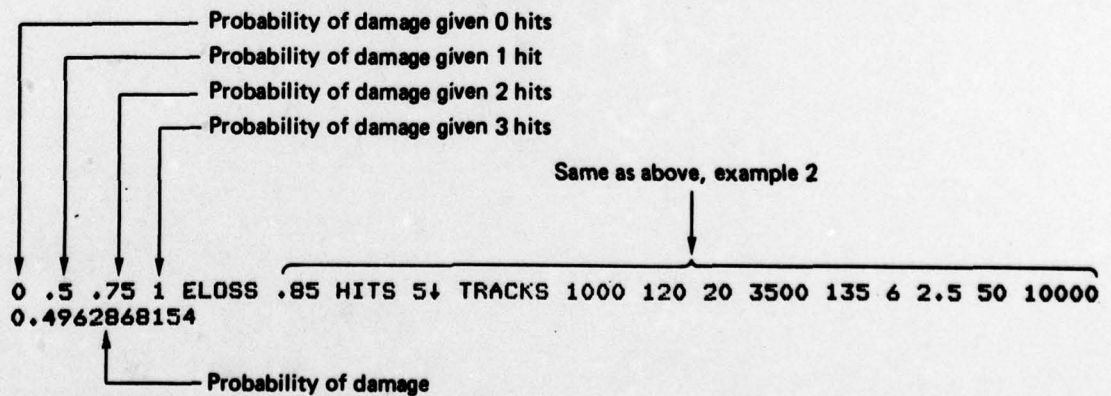
APPENDIX C
EXAMPLES OF APL PROGRAM USE

Example 4: Torpedo hits by multiple submarines (programs SUBS, HITS, and TRACKS)

```
    2 SUBS .85 HITS 5↓ TRACKS 1000 120 20 3500 135 6 2.5 50 10000
```

```
0.05691381074   0.1427361672   0.3100617983   0.2765858156   0.213702406
       0  0  0  0  0  0  0  0
```

Probability of 0, 1, 2, ..., 2×n hits with two submarines attacking from the same relative position.

Example 5: Expected losses from a multiple submarine attack; (programs ELOSS, SUBS, HITS, and TRACKS)

```
    0 0.5 0.75 1 ELOSS 3 SUBS 0.85 HITS 5↓TRACKS 1000 120 20 3500 135
6 2.5 50 10000
```

```
0.9251381545
```

Probability of damage with three submarines attacking from the same relative position.

Example 6: Expected losses from a multiple submarine attack; submarines in different relative positions (programs ELOSS, SUBS, HITS, TRACKS, and COMBINE)

```
    0 0.5 0.75 1 ELOSS ( 3 SUBS 0.85 HITS 5↓TRACKS 1000 120 20 3500 135
6 2.5 50 10000) COMBINE .9 HITS 5↓ TRACKS 1000 120 20 4000   60 5 1.5 6
0 8000
```

```
0.9900551757
```

Probability of damage with three submarines attacking from the same relative position and one submarine attacking from a different position with a different torpedo type.